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08MTP/MAU/MFD11

**First Semester M.Tech. Degree Examination, Dec.09/Jan.10**  
**Applied Mathematics**

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions.**

- 1 a. Find the binary form of the number 193. (06 Marks)  
 b. The number 3.1415927 is approximated as 3.1416. Find the following:  
 i) error ii) relative error and iii) number of significant digits of the approximation. (04 Marks)

c. If  $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -5 & 6 \end{bmatrix}$ ,  $B = \begin{bmatrix} 8 & 0 \\ 2 & 7 \\ -1 & 4 \end{bmatrix}$ , write Fortran program for the multiplication of the matrices A & B. (10 Marks)

- 2 a. Find the  $L_1$ ,  $L_2$ ,  $L_\infty$  and  $L_c$  norms of the matrix  $A = \begin{bmatrix} 4 & 3 \\ 7 & 6 \end{bmatrix}$  (06 Marks)  
 b. Find the solution of the following set of equations using the Gauss elimination method:  
 $2x_1 - x_2 + x_3 = 4$ ,  $4x_1 + 3x_2 - x_3 = 6$  and  $3x_1 + 2x_2 + 2x_3 = 15$  (06 Marks)

c. Find the inverse of the matrix  $A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 6 & -4 \\ 1 & -4 & 5 \end{bmatrix}$  using the relation  $[A]^{-1} = [U]^{-1}([U]^{-1})^T$ . (08 Marks)

- 3 a. Convert the eigen value problem  

$$\begin{bmatrix} 15000 & -5000 \\ -5000 & 5000 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \lambda \begin{bmatrix} 20 & 0 \\ 0 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$
 to a standard eigen value problem. (08 Marks)

- b. Find the Eigen values and the Eigen vectors of the matrix

$A = \begin{bmatrix} 12 & 6 & -6 \\ 6 & 16 & 2 \\ -6 & 2 & 16 \end{bmatrix}$  by using Jacobi method. (04 Marks)

- c. Generates the functions  $f_i(\lambda)$ ,  $i = 0, 1, 2, 3$  and 4 for the tridiagonal matrix

$A = \begin{bmatrix} 3 & -2 & 0 & 0 \\ -2 & 5 & -3 & 0 \\ 0 & -3 & 7 & -4 \\ 0 & 0 & -4 & 9 \end{bmatrix}$  (08 Marks)

- 4 a. Evaluate the mixed partial derivative  $\frac{\partial^4 f}{\partial x^2 \partial y^2}$  of the function  $f(x, y) = 2x^4 x^3$  using central differences at  $x = 1$  and  $y = 1$  with a step size  $\Delta x = \Delta y = 0.1$  (12 Marks)  
 b. Find the first three derivatives of  $f(x) = x^2 e^{-4x}$  and express Taylor's series expansion of  $f(x)$  at  $x = 1$ . (08 Marks)

7. Determine the value of the integral  $I = \int_a^b f(x)dx$ , where

$$f(x) = 0.84885406 + 31.51924706x - 137.66731262x^2 + 240.55831238x^3 - 171.45245361x^4 + 41.95066071x^5$$

with  $a = 0.0$  and  $b = 1.5$  using Simpson's 1/3 rule with different step sizes. (08 Marks)

8. Evaluate the integral  $I = \int_0^2 ye^{2y}dy$  using the Gauss-Legendre quadrature. (04 Marks)

9. Evaluate the integral  $I = \int_{z=0}^2 \int_{y=1}^4 \int_{x=-1}^3 5xy^3z^2 dx dy dz$  using the two-point Gauss-Legendre quadrature rule. (08 Marks)

10. Find the solution of the initial value problem  $y' = y + 2x - 1$ ;  $y(0) = 1$  in the interval  $0 \leq x \leq 1$  using Adams – Bashforth open formulas of order 2 through 6. (10 Marks)

11. Find the solution of the initial value problem.

$y' = -1 + 2x + y$ ;  $y(0) = 1$ , at  $x = 0.4$  using the fourth order Adams predictor-corrector method with  $h = 0.1$ . (10 Marks)

12. The differential equation governing the transverse deflection of a beam  $w(x)$  subjected to a distributed load,  $p(x)$  as shown in the Fig.7(a), is given as

$$\frac{d^2}{dx^2} \left( EI \frac{d^2 w}{dx^2} \right) = p(x)$$

where  $E$  = Young's modulus and  $I$  = area moment of inertia of the beam.

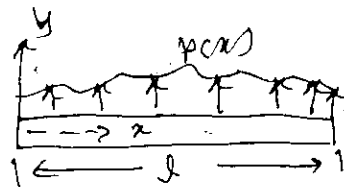


Fig.7(a)

13. Formulate the boundary value problem for a uniform beam i) fixed at the both ends and ii) simply supported at both ends. (08 Marks)

14. Explain the shooting methods. (12 Marks)

15. A metal rod of length 1 m is initially at  $70^\circ\text{C}$ . The steady-state temperature of the left and right ends of the rod are given as  $50^\circ\text{C}$  and  $20^\circ\text{C}$  respectively. Using  $\alpha^2 = 0.1 \text{ m}^2/\text{min}$ ,

$x = 0.2\text{m}$  and  $\Delta t = 0.3 \text{ min}$ , determine the temperature distribution in the rod for  $0 \leq t \leq 0.3$ , using Crank-Nicholson method. (10 Marks)

16. An aluminium plate of size  $0.3\text{m} \times 0.3\text{m}$  is initially at the temperature  $30^\circ\text{C}$ . If the adjacent faces of the plate are suddenly brought to  $120^\circ\text{C}$  and maintained at the temperature, derive the equations necessary for the determination of the time variation of temperature in the plate using the alternating direction implicit method.

Given  $K = 236 \text{ W/m-k}$ ,  $C = 900 \text{ J/kg-K}$ ,  $\rho = 2700 \text{ kg/m}^3$ ,  $\Delta x = \Delta y = 0.1 \text{ m}$ . (10 Marks)